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The softening of edge plasmons on lateral surfaces of coupled half-plane semiconductor superlattices

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Abstract. A hydrodynamic model has been used to study the magnetoplasmon modes on lateral surfaces of two coupled half-plane semiconductor superlattices. The numerical calculation result predicts that the coupled localised edge modes may be softened because of the coupling between two lateral surfaces. The new softened surface plasmon mode in two coupled half-bulks is obtained. Several special cases are also discussed.

Recently, the collective excitations of low-dimensional electron gases have attracted much attention both theoretically and experimentally. A kind of edge magnetoplasmon mode that propagates along the boundary of the 2D electron fluid has been found [1–7]. The frequency of this edge mode varies inversely with the magnetic field in the large-field limit.

However, in [8] an analysis was reported of intra-sub-band surface plasmon modes on the lateral surface of a half-plane semiconductor superlattice; these were called the 'edge modes' of such a system. The edge magnetoplasmon modes on a lateral surface of a half-plane superlattice with a complex unit cell and coupled edge magnetoplasmon modes in an electron fluid confined to a plane with a channel have been studied in [6, 7]. These excitations are peculiar in that they are free of Landau damping and could prove of interest in the field of surface wave devices [9], in which the effect of interactions between two planes on the collective excitation modes is a very realistic problem.

A hydrodynamic model will be used here to study the magnetoplasmon modes of two coupled half-plane superlattices. For simplicity, we use a model in which periodic arrays of 2D electron layers are stacked along the z direction, and the electron layers are located in the spaces $x < 0$ (region 1) and $x > a$ (region 2) of distance a apart and embedded in a semiconductor background of dielectric constant ε_s. The external magnetic field is along the z direction perpendicular to the half-planes.

The principal problem of interest is the self-consistent oscillation of a charge-compensated 2D electron gases with the layers situated at $x < 0$, and $x > a$, placed in a perpendicular magnetic field $B\hat{z}^0$. Consider a rigid positive background with charge density en_0 and a compressible electron fluid with number density $n_0 + n$. Let $n_j(r, t)$ and $V_j(r, t)$ denote, respectively, the small fluctuation in the electron surface density and the electron velocity field in the plane of the jth layer. These amplitudes satisfy the equation of continuity, Euler's equations and Poisson's equation:

$$-\mathrm{i}_{\omega} n_j + n_0(\partial v_{jx}/\partial x + \mathrm{i}k v_{jy}) = 0 \tag{1}$$

$$-\mathrm{i}_{\omega} v_{jx} + (s^2/n_0)\partial n_j/\partial x - (e/m^*)\partial\varphi/\partial x + \omega_c v_{jy} = 0 \tag{2}$$

$$-\mathrm{i}_{\omega} v_{jy} + \mathrm{i}ks^2(n_j/n_0) - \mathrm{i}k(e/m^*)\varphi - \omega_c v_{jx} = 0 \tag{3}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - k^2\right)\varphi(x, z) = \frac{4\pi e}{\varepsilon_s}\sum_{j=-\infty}^{+\infty} n_j(x)\delta(z - z_j)[\theta(-x) + \theta(x - a)] \tag{4}$$

where φ is the electrostatic potential and ω_c is the cyclotron frequency. θ is the step function, and s is an effective compressional wave speed. Here, since the system is translationally invariant along the y direction, the solution may be taken as a plane wave of the form $\exp(\mathrm{i}ky - \mathrm{i}\omega t)$, with amplitudes which depend on x and z. It is convenient to consider k positive, while ω_c can take either sign. A Fourier transform in x of equation (4) gives the ordinary differential equation

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - (k^2 + k_x^2)\right)\varphi(k_x, z) = \frac{4\pi e}{\varepsilon_s}\sum_{j=-\infty}^{+\infty} n_j(k_x)\delta(z - z_j) \tag{5}$$

where $n_j(k_x)$ is the Fourier transform of $n_j(x)[\theta(-x) + \theta(x - a)]$. Its solution can be written as

$$\varphi(k_x, z) + \frac{2\pi e}{\varepsilon_s}\sum_{j=-\infty}^{+\infty} n_j(k_x)k^{-1}\exp(-k'|z - z_j|) = 0 \tag{6}$$

where $k' = (k^2 + k_x^2)^{1/2}$. The inverse Fourier transform then gives a non-local integral relation between the electrostatic potential in the lth plane and the corresponding charge density

$$\varphi(x, z_1) + \frac{4\pi e}{\varepsilon_s}\sum_{j=-\infty}^{+\infty}\int \mathrm{d}x'\, L_j(x - x')n_j(x')[\theta(-x') + \theta(x' - a)] = 0 \tag{7}$$

where

$$L_j(x) = \int \mathrm{d}k_x \exp(\mathrm{i}k_x x)[(2k')^{-1}\exp(-k'|z_l - z_j|)]. \tag{8}$$

In principle, such an integral equation can be solved using the Wiener–Hopf technique [10]. Using the usual Bloch condition

$$n_j(x') = A(x')\exp(\mathrm{i}q_z jd) \tag{9}$$

we have

$$\varphi(x, z_l) + \frac{2\pi e}{\varepsilon_s}\int \mathrm{d}k_x \exp(\mathrm{i}k_x x)[k'^{-1}A(k_x)S(k_x, k, q_z)] = 0 \tag{10}$$

where

$$S(k_x, k, q_z) = \sinh(k'd)/[\cosh(k'd) - \cos(q_z d)] \tag{10a}$$

$$A(k_x) = \int \mathrm{d}x' \exp(-\mathrm{i}k_x x')A(x'). \tag{10b}$$

They are independent of the layer label. Combination of equations (7) and (10) yields the Fourier component of the exact kernel about equation (7):

$$L(k_x) = [2(k_x^2 + k^2)^{1/2}]^{-1} S(k_x, k, q_z). \tag{11}$$

We introduce here the approximation method used in [5] which we think remains applicable and will be seen to work with equal ease [6, 7], then we obtain

$$L_0(k_x) = kf(k, q_z)/[2k^2 + k_x^2 g(k, q_z)] \tag{12}$$

where

$$g(k, q_z) = 1 - [k/f(k, q_z)][\mathrm{d}f(k, q_z)/\mathrm{d}k]. \tag{13}$$

Function $g(k, q_z)$ characterises the screening correction for edge plasmons, and $f(k, q_z) = S(k_x = 0, k, q_z)$. $L(k_x)$ and $L_0(k_x)$ have the same first two terms in a power series about $k_x^2 = 0$. The inverse Fourier transform of equation (12) gives the approximate kernel

$$L_0(x) = 2^{-1} f(k, q_z)(2g)^{-1/2} \exp[-(2/g)^{1/2} k|x|]. \tag{14}$$

The problem can be reduced to a pair of effective localised Poisson's equations

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} - \frac{2k^2}{g}\right) \varphi_{1,2}(x, z_i) = \frac{4\pi ek}{\varepsilon_s} fg^{-1} \sum_{j=-\infty}^{+\infty} n_j(x) \qquad x<0 \quad \text{or} \quad x>a \tag{15}$$

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} - \frac{2k^2}{g}\right) \varphi_3(x, z_i) = 0 \qquad 0<x<a. \tag{16}$$

The remaining steps in the solution are identical with those in [6]. When equations (1)–(3), (15) and (16) are combined with the boundary conditions that φ and $\partial\varphi/\partial x$ are continuous and that v_x vanishes there, together with the suitable boundary behaviour $|x| \to \infty$, this procedure gives the dispersion relation

$$\begin{aligned} &D^4\omega^2\{2(2/g)^{1/2} C \sinh[(2/g)^{1/2}ka] + C^2 \sinh[(2/g)^{1/2}ka] + (2/g) \sinh[(2g)^{1/2}ka]\} \\ &\quad - 4(2/g)^{1/2} D^2 \omega_k^2 \omega^2 (f/g)\{C \cosh[(2/g)^{1/2}ka] \\ &\quad + (2/g)^{1/2} \cosh[(2/g)^{1/2}ka]\} + 4\omega_k^4 (f/g)\{(2/g)\omega^2 \sinh[(2/g)^{1/2}ka] \\ &\quad - \omega_c^2 \sinh[(2/g)^{1/2}ka]\} = 0 \end{aligned} \tag{17}$$

where $\omega_k^2 = 2\pi n_0 e^2 k/m^*$ is the bulk 2D plasma frequency, and

$$D^2 = 2\omega_k^2(f/g) + (\omega_c^2 - \omega^2) \tag{18}$$

$$C^2 = 2[(\omega_k^2(f/g) + (1/g)(\omega_c^2 - \omega^2)]/[2\omega_k^2(f/g) + (\omega_c^2 - \omega^2)]. \tag{19}$$

An additional set of roots are given by $\omega^2 = \omega_c^2$ (spurious result of the approximation method) and $\omega^2 = 2\omega_k^2(f/g) + \omega_c^2$ (corresponding to the bulk continuum when $a \to 0$) [6].

It is easy to find several well known special results.

(i) When $a \rightarrow 0$, we obtain from equation (17)

$$\omega^2 = \omega_c^2 + 2\omega_k^2(f/g) \tag{20}$$

which corresponds to the bulk continuum. For $d \rightarrow 0$, equation (20) gives the dispersion relation for a three-dimensional electron gas (3DEG):

$$\omega^2 = \omega_c^2 + \Omega_p^2 + O(q^2) \tag{20a}$$

where $\Omega_p^2 = 4\pi n_0 e^2/\varepsilon_s m^* d$ is the 3D plasma frequency. When d is finite, we let $g = 2$, corresponding to the exact result [1] and then obtain the dispersion relation referring to the superlattice:

$$\omega^2 = \omega_c^2 + \omega_k^2 \sinh(kd)/[\cosh(kd) - \cos(q_z d)]. \tag{20b}$$

Furthermore, for $d \rightarrow \infty$, equation (20*b*) reduces to the dispersion relation of a 2DEG:

$$\omega^2 = \omega_c^2 + \omega_k^2. \tag{20c}$$

(ii) When $a \rightarrow \infty$, we obtain from equation (17) for a finite value of d

$$\begin{aligned} \omega_+ &= 2^{1/2}(2+g)^{-1}\,\mathrm{sgn}(\omega_c)\{[(2+g)f\omega_k^2 + g\omega_c^2]^{1/2} + g^{1/2}|\omega_c|\} \\ \omega_- &= -2^{1/2}(2+g)^{-1}\,\mathrm{sgn}(\omega_c)\{[(2+g)f\omega_k^2 + g\omega_c^2]^{1/2} - g^{1/2}|\omega_c|\} \end{aligned} \tag{21a}$$

which is related to a half-plane superlattice. When $d \rightarrow 0$, equation (21*a*) reduces to the dispersion relation of an EG in a half-bulk:

$$\begin{aligned} \omega_+ &= 2^{-1}\,\mathrm{sgn}(\omega_c)\{[2\Omega_p^2 + \omega_c^2 + O(q^2)]^{1/2} + |\omega_c|\} \\ \omega_- &= -2^{-1}\,\mathrm{sgn}(\omega_c)\{[2\Omega_p^2 + \omega_c^2 + O(q^2)]^{1/2} - |\omega_c|\}. \end{aligned} \tag{21b}$$

If $d \rightarrow \infty$, on the contrary, equation (21*a*) reduces to the dispersion relation of an EG in a half-plane

$$\begin{aligned} \omega_+ &= (\tfrac{2}{9})^{1/2}\,\mathrm{sgn}(\omega_c)\{[3\omega_k^2 + \omega_c^2]^{1/2} + |\omega_c|\} \\ \omega_- &= -(\tfrac{2}{9})^{1/2}\,\mathrm{sgn}(\omega_c)\{[3\omega_k^2 + \omega_c^2]^{1/2} - |\omega_c|\}. \end{aligned} \tag{21c}$$

(iii) When a is finite, for $d \rightarrow \infty$, equation (17) leads to the result corresponding to an EG in two coupled half-planes with a channel given in [6].

Equation (17) can be readily solved by the numerical method to give the desired edge plasmon dispersion relation and magnetic field dependence of the frequency of edge magnetoplasmons. In general, we have two branches of coupled modes, as shown in figure 1. (The other two band edges corresponding to $q_z d = \pi$ has not been shown for $\omega_c \neq 0$.) When $\omega_c = 0$, there are two branches of modes due to coupling (a is finite). In the absence of a magnetic field, it should be pointed out that, for the strong screening $kd \ll 1$, the frequency of the anomalous edge mode rapidly decreases when a becomes small; this is called the 'softened' plasmon mode and is very different from that given in [6]. The 'softened' plasmon mode can be attributed to the dramatic enhancement of complete Coulomb screening due to the strong coupling between different layers as a decreases. Under the presence of the magnetic field the symmetry with respect to the $+y$ and $-y$ directions is broken, and then the spectrum of the edge plasmon mode will split. However, when there are two coupled half-plane superlattices, the possible combinations of the two directions are $(+y, +y)$, $(-y, -y)$, $(+y, -y)$ and $(-y, +y)$. Of

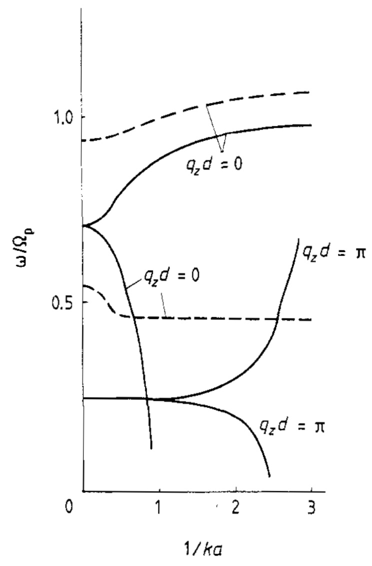

Figure 1. The coupling strength dependence of the coupled edge mode in units of Ω_p for $kd = 0.5$ and different external magnetic fields: ——, no external magnetic field, $\omega_c = 0$; - - -, $\omega_c/\Omega_p = 0.4$ and $q_zd = 0$. The other two band edges corresponding to $q_zd = \pi$ are not shown for the broken lines.

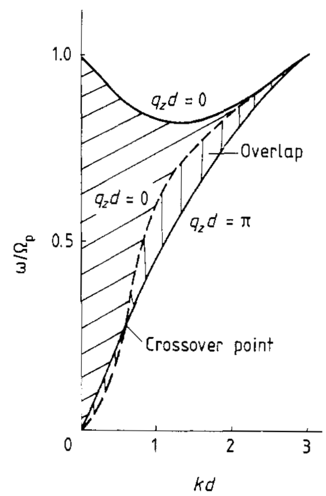

Figure 2. Dispersion relation of edge magnetoplasmon modes for coupled half-plane superlattices. The parameters are as follows: $\omega_c/\Omega_p = 0$; $a/d = 2.0$.

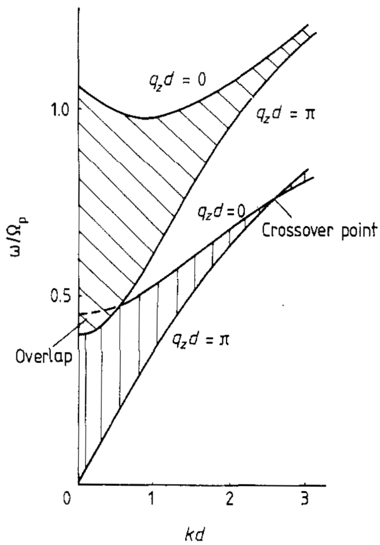

Figure 3. Dispersion relation of edge magnetoplasmon modes for coupled half-plane superlattices. The parameters are as follows: $\omega_c/\Omega_p = 0.4$, $a/d = 2.0$.

these, $(+y, +y)$ and $(-y, -y)$ are equivalent, and so are $(+y, -y)$ and $(-y, +y)$. These two kinds of combination correspond to two modes of edge magnetoplasmons. The existence of a magnetic field reduces the softening of anomalous edge mode. Also, the tops and bottoms of the two branches are interchanged.

From figure 2, we know that for a definite geometrical factor ratio a/d, if $\omega_c = 0$, there are two branches of coupled modes, which overlap each other owing to coupling. The lower branch tends to zero as $kd \rightarrow 0$ and so does the lower edge of the upper

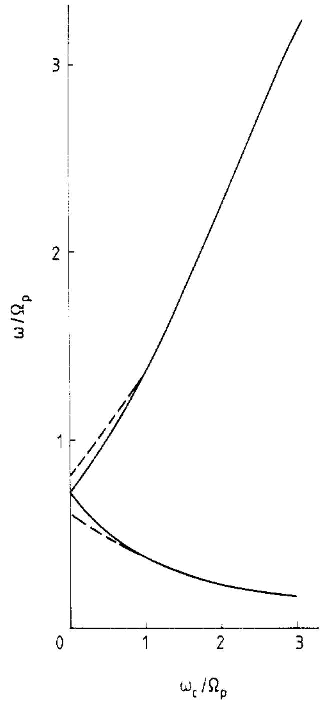

Figure 4. The magnetic field dependence of the edge magnetoplasmon modes in units of Ω_p for $q_z d = \pi$, $kd = 0.5$ and different coupling strengths: ——, a is infinite; ----, weak coupling, $ka = 2.0$. The other two band edges corresponding to $q_z d = \pi$ are not shown for the full and broken lines.

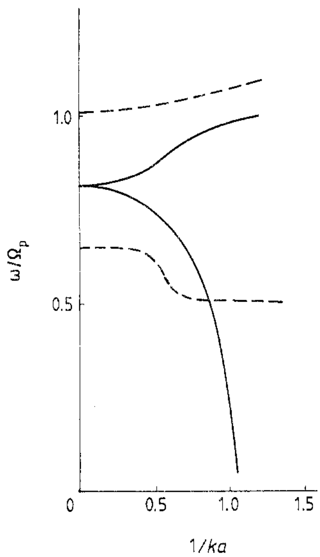

Figure 5. The coupling strength dependence of the coupled edge modes in two coupled half-bulks ($d \to 0$) in units of Ω_p for different external magnetic fields: ——, no external magnetic field, $\omega_c = 0$; ---, $\omega_c/\Omega_p = 0.4$.

branch. The existence of a magnetic field raises the lower edge of upper branch in the strong screening region $kd \ll 1$, as shown in figure 3. If the magnetic field is large enough, a gap will exist between two branches. As the magnetic field increases, so does the gap. Moreover, the top and bottom of the lower branch are interchanged.

Figure 4 presents the magnetic field dependence of the edge magnetoplasmon modes for different coupling strengths $(ka)^{-1}$. When $\omega_c = 0$, the existence of coupling leads to cancellation of the degeneracy of the magnetoplasmon mode. As the coupling strength increases, the splitting increases also. The normal edge mode increases with increasing magnetic field $\boldsymbol{B}$ as expected, while the anomalous edge mode intensity is proportional to $\boldsymbol{B}^{-1}$ in the large-field limit. The other two band edges corresponding to $q_z d = \pi$ are not shown here.

It is very interesting to study the following special case. If a is finite, for $d \to 0$, the dispersion relation of the coupled surface modes of two half-bulks is given by

$$4R^4\omega^2 \sinh(ka) - 4R^2\omega^2\Omega_p^2 \cosh(ka) + \Omega_p(\omega^2 - \omega_c^2)\sinh(ka) = 0 \qquad (22a)$$

with

$$R^2 = \Omega_p^2 + (\omega^2 - \omega_c^2) \qquad (22b)$$

which is presented in figure 5. In this case, the screening becomes stronger, and the splitting of the two branches becomes smaller. The band width is zero in this case. From this we can predict the existence of new coupled surface modes in such a system. Similar to the result in figure 1, when $\omega_c = 0$, softening of the anomalous surface mode may occur. The enhancement of complete Coulomb screening weakens the interaction between electrons localised at two surfaces, so that the frequency of the anomalous surface mode is decreased. The existence of a magnetic field also reduces the softening of the anomalous surface mode.

Although it is difficult to estimate the accuracy of the present approximation, an exact study of such a system, which needs considerable computation and will be given in a separate paper, can show that this approach provides a good qualitative fit to the field dependence of the anomalous edge modes if the magnetic field is not very large, and it also is reasonably good for the normal edge modes of zero-field values. The softened plasmon mode in two coupled half-plane superlattices and the softened surface mode in two coupled half-bulks are attractive ideas which have not previously been reported. Moreover, it leads us to another approach to obtain two coupled branches conveniently with adjustable separation between them by using coupled half-plane superlattice structure in the presence of a magnetic field, instead of using a complex unit-cell structure.

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